Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Engineering 29 (2012) 785 – 790
Procedia Engineering
www.elsevier.com/locate/procedia

2012 International Workshop on Information and Electronics Engineering (IWIEE 2012)

# The Feasibility Analysis of Aerial Moving Target Location Based on TDOA/FDOA/DSF Measurements

Bing Deng[a]*, Jinyu Xiong[a], Changxiong Xia[a]

[a]*Science and Technology On Blind Signal Processing Laboratory, Chengdu, China*

**Abstract**

In this paper, Doppler-shift frequency measurements are introduced into the Dual-Satellite Geolocation System to implement location for aerial moving target. The location model is established first, and then the rank of the observation matrix is proposed as standards of location observability, and two typical motion models, the uniform linear motion and the maneuvering turn motion are analysed and their observation matrix are given. Finally, simulation results presente a conclusion that Dual-satellite geolocation system can implement location effectively for aerial moving target by adding the Doppler-shift frequency measurements of single satellite under some restrictions.



*Keywords:* Dual-satellite geolocation; TDOA; FDOA; Doppler-shift frequency; moving target; observability

## 1. Introduction

As an effective location technique, Dual-satellite geolocation takes much attention lately. This system estimates the location of a stationary emitter or slow moving target using time differences of arrival (TDOA) and frequency differences of arrival (FDOA) measurements of a signal as retransmitted by the two adjacent geostationary satellites in the area of visibility of both satellites [1].However, there is no public report about aerial moving target location by using this system. So, in this paper, the DSF (Doppler-shift frequency) measurements of single satellite are adding to this system to implement the aerial moving target location. So, this paper mostly analyse the observability of aerial moving target based on Dual-satellite geolocation system.

* Corresponding author. Tel.: 086-28-8659-5381; fax: 086-28-8659-5360.
*E-mail address*: dengbing3832005022@163.com.


doi:10.1016/j.proeng.2012.01.042

## 2. System Location Models

### 2.1. Dual-satellite TDOA/FDOA location model

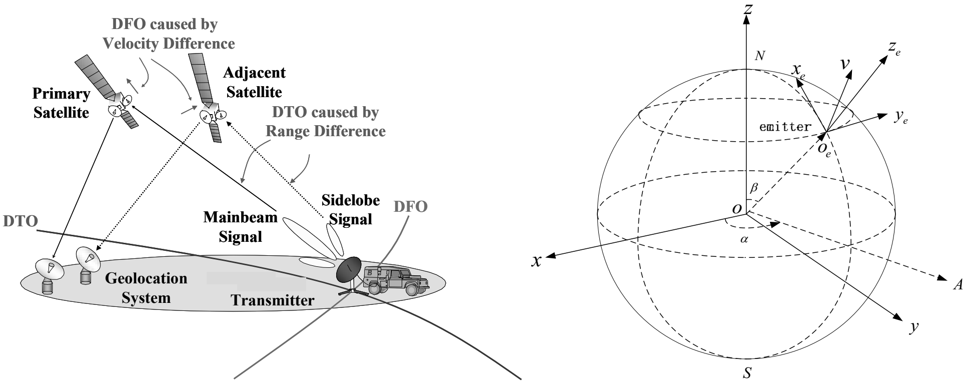


Fig.1.(a) Dual-Satellite geolocation principle;(b)The coordinates transform sketch map

In Dual-satellite geolocation system, because of the signals received by the two different satellites have different transmit paths, there are a DTO (differential time offset) and a DFO (differential frequency offset) between the two uplink signals, caused by velocity difference and range difference of both satellites [2]. Fig 1.(a) illustrates the principle of the dual-satellite geolocation system.

The position of target is $\mathbf{r}_1$ , while the primary satellite and adjacent satellite's positions are $\mathbf{r}_{s1}$ and $\mathbf{r}_{s2}$ respectively. From the location geometry with the uplink paths[3], the location equations can be given by

$$\begin{cases} dto = (\left\|\mathbf{r}_{s1}-\mathbf{r}_1\right\| - \left\|\mathbf{r}_{s2}-\mathbf{r}_1\right\|)/c \\ dfo = \dfrac{f_0}{c}\cdot\left[\mathbf{v}_{s1}\cdot\dfrac{\mathbf{r}_{s1}-\mathbf{r}_1}{\left\|\mathbf{r}_{s1}-\mathbf{r}_1\right\|} - \mathbf{v}_{s2}\cdot\dfrac{\mathbf{r}_{s2}-\mathbf{r}_1}{\left\|\mathbf{r}_{s2}-\mathbf{r}_1\right\|}\right] \\ \mathbf{r}_1\mathbf{r}_1^{\mathrm{T}} = R^2 \end{cases} \tag{1}$$

Where the $c$ is the velocity of light, $c = 2.998\times10^8 m/s$ .The $f_0$ is the center frequency of the uplink signal. $R$ denotes the earth radius.

Fig 1.(b) illustrates the coordinates transformation sketch map from the Station's Coordinates to the Earth Center Fixed Coordinates, where $x_e y_e z_e$ denotes the station coordinates, $xyz$ denotes the earth center coordinates, $\alpha$ and $\beta$ are the longitude and latitude in $xyz$ , respectively.

So the coordinates transformation matrix form $x_e y_e z_e$ to $xyz$ is:

$$M = \begin{bmatrix} \cos(\pi/2+\alpha) & -\sin(\pi/2+\alpha)\cos(\pi/2-\beta) & \sin(\pi/2+\alpha)\sin(\pi/2-\beta) \\ \sin(\pi/2+\alpha) & \cos(\pi/2+\alpha)\cos(\pi/2-\beta) & -\cos(\pi/2+\alpha)\sin(\pi/2-\beta) \\ 0 & \sin(\pi/2-\beta) & \cos(\pi/2-\beta) \end{bmatrix} \tag{2}$$

### 2.2. Aerial moving target location models

The primary satellite receives the mainbeam signal, while the adjacent satellite which is adjacent to the primary satellite receives the sidelobe signal in Dual-satellite geolocation system. The parameters can be used are DTO and DFO, while the target's altitude $h$ is unknown. Then, the DSF measurements of the primary satellite are introduced to the parameters. So, there are 3 location parameters with 6 unknown quantities. Otherwise, this can be solved by adding measurement times, namely adding the different times' DTOs and DFOs and DSFs. So, the location equation is listed below:

$$\begin{cases} dto_k = (\|\mathbf{r}_{\mathbf{s1}k} - \mathbf{r}_{\mathbf{1}k}\| - \|\mathbf{r}_{\mathbf{s2}k} - \mathbf{r}_{\mathbf{1}k}\|)/c \\ dfo_k = \dfrac{f_0}{c} \cdot \left[ (\mathbf{v}_{\mathbf{s1}k} - \mathbf{v}_{\mathbf{1}k}) \cdot \dfrac{\mathbf{r}_{\mathbf{s1}k} - \mathbf{r}_{\mathbf{1}k}}{\|\mathbf{r}_{\mathbf{s1}k} - \mathbf{r}_{\mathbf{1}k}\|} - (\mathbf{v}_{\mathbf{s2}k} - \mathbf{v}_{\mathbf{1}k}) \cdot \dfrac{\mathbf{r}_{\mathbf{s2}k} - \mathbf{r}_{\mathbf{1}k}}{\|\mathbf{r}_{\mathbf{s2}k} - \mathbf{r}_{\mathbf{1}k}\|} \right] \\ fd_k = \dfrac{f_0}{c} \cdot (\mathbf{v}_{\mathbf{s1}k} - \mathbf{v}_k) \cdot \dfrac{\mathbf{r}_{\mathbf{s1}k} - \mathbf{r}_k}{\|\mathbf{r}_{\mathbf{s1}k} - \mathbf{r}_k\|} \end{cases} \tag{3}$$

Here, $fd$ is the DSF of the primary satellite, subscript $k$ denotes the measurement times, and $k = 1, 2, 3 \dots$.

There, Presumes the state vector of the uniform linear motion is $X(k)$, which is composed of the target's position and velocity. Where

$$\begin{cases} X(k) = [x_k \quad y_k \quad z_k \quad v_x \quad v_y \quad v_z]^T \\ X(k) = \phi X(k-1) \end{cases} \tag{4}$$

*2.2.1 the uniform linear motion*

About the uniform linear motion, The $\mathbf{r_k}$ denotes target's position at $t_k$ set, $\mathbf{v}$ is the velocity vector.

$$\mathbf{r_k} = \mathbf{r_{k+1}} + \mathbf{v} \cdot (t_k - t_{k-1}) \tag{5}$$

$$\phi_{k-1,k} = \begin{bmatrix} 1 & 0 & 0 & T_{k-1,k} & 0 & 0 \\ 0 & 1 & 0 & 0 & T_{k-1,k} & 0 \\ 0 & 0 & 1 & 0 & 0 & T_{k-1,k} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Here, $T_{k-1,k} = t_k - t_{k-1}$ is the measurement interval.

*2.2.2 the maneuvering turn motion*

For the maneuvering turn motion, presumes the target is turning in the $x_e o_e y_e$ plane which illustrated in Fig 1.(b). Also, the state vector of this motion is $X(k)$, which is composed of the target's position and velocity. Then

$$\phi = \begin{bmatrix} 1 & 0 & 0 & -\dfrac{\sin(\omega T)}{\omega} & -\left(\dfrac{1-\cos(\omega T)}{\omega}\right) & 0 \\ 0 & 1 & 0 & -\left(\dfrac{1-\cos(\omega T)}{\omega}\right) & \dfrac{\sin(\omega T)}{\omega} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\omega T) & -\sin(\omega T) & 0 \\ 0 & 0 & 0 & \sin(\omega T) & \cos(\omega T) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$T$ is the measurement interval, $\omega$ is a constant ,and denotes the angle speed of turn motion.

## 3. Observability Analyse

The passive location of aerial moving target problem is an intrinsic nonlinear problem, and nonlinear observability analysis theory was first published in the work of [4] .

### *3.1. Theory of Nonlinear Observability*

The state vector is $x(k)$, measurement is $y(k)$ [5].

$$\begin{cases} \dot{x}(k) = f(x(k),k) \\ y(k) = h(x(k),k) \end{cases} \tag{8}$$

$x(k_0) = x_0$ denotes system's initial value [7] shows the following theorem for the observability of (9).

**Theorem:** If $f(x(k),k) = g^0(x(k)) + \sum g^i(x(k))k_i$ , where $x(k)$ is a vector of $n$ state variables occupying an open subset $S \in R^n$ . $g^0()\ldots g^i()$ are $n$ dimensional vector analytical functions in $S$ , the output $h(.)$ is an analytic function of $R^n$ in $S$ and $k$ is an analytic function of time having distinct scalar controls $k_i$ ,then (9) is locally observable if the matrix $\Phi$ given below has full rank(i.e rank $n$ ).

$$\Phi(d,f,h,n) = [(dL_f^0 h) \quad (dL_f^1 h) \quad \cdots \quad (dL_f^{n-1} h)]^T \tag{9}$$

where $h(.) = [h_1(.) \quad h_2(.) \quad \cdots \quad h_p(.)]^T$ . $L_f^m g_i$ is the $m^{th}$ order Lie derivative of $g_i(.)$ with respect to the function $f(.)$ and operator $d$ denote the gradient operator with respect to $x$ . The proof for this theorem is given in [6].

### *3.2. Observability analyse based on Dual-satellite system*

After 2 times' measurement for uniform linear motion, the observable matrix is:

$$\mathbf{\Gamma_1} = \begin{bmatrix} -(\mathbf{u}_{21} - \mathbf{u}_{11})/c & \mathbf{0} \\ \left(\nabla v_{211}(\mathbf{r}) - \nabla v_{obj1}(\mathbf{r})\right)\cdot f_0/c & -(\mathbf{u}_{21} - \mathbf{u}_{11})\cdot f_0/c \\ \nabla v_{mf1}(\mathbf{r})\cdot f_0/c & \mathbf{u}_{11}\cdot f_0/c \\ -(\mathbf{u}_{22} - \mathbf{u}_{12})/c & -(t_2 - t_1)\cdot(\mathbf{u}_{22} - \mathbf{u}_{12})/c \\ \left(\nabla v_{212}(\mathbf{r}) - \nabla v_{obj2}(\mathbf{r})\right)\cdot f_0/c & \left[-(t_2 - t_1)\cdot\left(\nabla v_{212}(\mathbf{r}) - \nabla v_{obj2}(\mathbf{r})\right) - (\mathbf{u}_{22} - \mathbf{u}_{12})\right]\cdot f_0/c \\ \nabla v_{mf2}(\mathbf{r}) f_0/c & \left((t_2 - t_1)\cdot\nabla v_{mf2}(\mathbf{r}) - \mathbf{u}_{12}\right) f_0/c \end{bmatrix} \tag{10}$$

Where: $\mathbf{0} = [0 \quad 0 \quad 0]$ , $\mathbf{u}_{2k} - \mathbf{u}_{1k} = \dfrac{\mathbf{r}_{s2k} - \mathbf{r}_k}{\|\mathbf{r}_{s2k} - \mathbf{r}_k\|} - \dfrac{\mathbf{r}_{s1k} - \mathbf{r}_k}{\|\mathbf{r}_{s1k} - \mathbf{r}_k\|}$

$$\nabla v_{21k}(\mathbf{r}) = \frac{-1}{\|\mathbf{r}_{s2k} - \mathbf{r}_k\|}\left[\mathbf{v}_{s2k} - (\mathbf{v}_{s2k}\cdot\mathbf{u}_{2k})\mathbf{u}_{2k}\right] + \frac{1}{\|\mathbf{r}_{s1k} - \mathbf{r}_k\|}\left[\mathbf{v}_{s1k} - (\mathbf{v}_{s1k}\cdot\mathbf{u}_{1k})\mathbf{u}_{1k}\right]$$

$$\nabla v_{objk}(\mathbf{r}) = \frac{-1}{\|\mathbf{r}_{s2k} - \mathbf{r}_k\|}\left[\mathbf{v} - (\mathbf{v}\cdot\mathbf{u}_{2k})\mathbf{u}_{2k}\right] + \frac{1}{\|\mathbf{r}_{s1k} - \mathbf{r}_k\|}\left[\mathbf{v} - (\mathbf{v}\cdot\mathbf{u}_{1k})\mathbf{u}_{1k}\right]$$

Here, $k$ is the measurement set, $k = 1,2,3...$ Obviously, when $\mathbf{\Gamma_1}$ is of rank 6,which is equal to $\det\mathbf{\Gamma_1} \neq 0$ ,the uniform linear motion of aerial moving target is observable by using Dual-satellite system, or else, it should increases measurement sets to let target be observable. There, the observable matrixes of much more than 2 times are not listing.

After 2 times' measurement for maneuvering turn motion, the observable matrix is:

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} -(\mathbf{u}_{21}-\mathbf{u}_{11})\cdot M/c & \mathbf{0} \\ (\nabla v_{211}(\mathbf{r})-\nabla v_{obj1}(\mathbf{r}))\cdot f_0/c\cdot M & -(\mathbf{u}_{21}-\mathbf{u}_{11})\cdot f_0/c\cdot M \\ \nabla v_{mf1}(\mathbf{r})\cdot f_0/c\cdot M & \mathbf{u}_{11}\cdot f_0/c\cdot M \\ [-(\mathbf{u}_{22}-\mathbf{u}_{12})\cdot M/c & \mathbf{0}]\phi \\ [(\nabla v_{212}(\mathbf{r})-\nabla v_{obj2}(\mathbf{r}))\cdot f_0/c\cdot M & -(\mathbf{u}_{22}-\mathbf{u}_{12})\cdot f_0/c\cdot M]\phi \\ [\nabla v_{mf2}(\mathbf{r})\cdot f_0/c\cdot M & \mathbf{u}_{12}\cdot f_0/c\cdot M]\phi \end{bmatrix} \tag{11}$$

Here, the symbols' express the same meaning in equation (10).Obviously, when $\boldsymbol{\Gamma}_2$ is of rank 6, namely $\det\boldsymbol{\Gamma}_2 \neq 0$ , the maneuvering turn motion is observable by Dual-satellite system.

## 4. Simulations

Simulations are carried out based on the applying demands of passive location for an aerial moving target. The receivers are two different geostationary satellites. The ephemeris is shows in table 1.

Tab.1 the positions of two geostationary satellites

| **coordinates** | X(m) | Y(m) | Z(m) | $V_X$(m/s) | $V_Y$(m/s) | $V_Z$(m/s) |
|---|---|---|---|---|---|---|
| **satellite1** | -18474807.196 | 37900357.993 | 248121.483 | 7.908 | 2.385 | 224.496 |
| **satellite2** | -13727309.558 | 39867005.798 | -4713.916 | 0.001 | 0.001 | 3.225 |

For the uniform linear motion, assumes the target located the (121°E,48°N)，where is in the area of visibility of both satellites. The module of the target's velocity is 150m/s, and the initial flight's pitching angle is 0°, the initial flight's azimuth angle is 37°.

The initial parameters of the maneuvering turn motion are the same as the former, and the angle speed $\omega = 0.03 rad/s$ .So, the radius and the period of the turn motion are 5km and 209.44s, respectively.

Fig 2.(a) and Fig 2.(b) show the ranks of $\boldsymbol{\Gamma}_1$ in equation (10) and the rank of $\boldsymbol{\Gamma}_2$ in equation (11),with the different measure intervals and times, respectively.

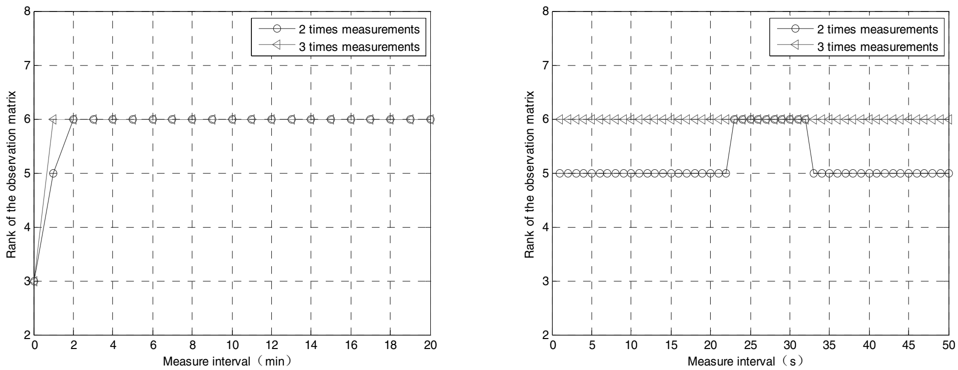


Fig.2.(a) The rank of the observation matrix for the uniform linear motion; (b) The rank of the observation matrix for the maneuvering turn motion

Though the computer simulation examples above, we can see that:

First point, the uniform linear motion and he maneuvering turn motion for aerial moving target are observable by using Synchronous Dual-satellite system with adding the DSFs' information after some times' measurements.

Second point, the uniform linear motion with 2 times measurements needs about 4 minutes to locate, and upwards 3 times measurements, the time can be reduced to about 3 minutes. The maneuvering turn motion can be observable with 2 times measurements in some intervals. With no less than 3 times measurements, about 3s, this motion can be observable.

## 5. Conclusion

In this paper, though introducing the Doppler-shift frequency measurements of the primary satellite into the Dual-satellite geolocation system, the observability of aerial moving target location for a uniform linear motion and maneuvering turn motion are analysed. Although numerous simulation tests were conducted during the study, only two representatives of results have been presented. Nevertheless, these results are sufficient to demonstrate the aerial moving target can be located based on TDOA, FDOA and DSF measurements under some restrictive conditions.

## References

[1] Wen-zhong Qu, etc., "Algorithm of position calibrator for satellite interference location", Chinese Journal of Radio Science, October 2005, Vol.20, No.3, pp.342-346.

[2] Zheng-bong Sun, Shang-fu Ye, "Satellite interference location using cross ambiguity function",Chinese Journal of Radio Science, October 2004,Vol.19, No.5, pp.525-529.

[3] Hang Yan, Jin kun Cao, and Lei Chen, "Study on Location Accuracy of Dual-Satellite Geolocation System", Proceedings of 2010 the 10th IEEE International Conference on Signal Processing . Beijing CHINA, Oct. 24-28 2010.

[4] L.D.L. Perera etc., "On the Linear and Nonlinear Observability Analysis of the SLAM Problem", Proceedings of the 2009 IEEE International Conference on Mechatronics. Malaga, Spain, April 2009.

[5] R Hermann and A. 1. Krener. Nonlinear Controllability and Observability. IEEE Trans.on Automatic Control. 22(5):728-740.October 1977.

[6] M. Anguelova. Observability and identifiably of nonlinear systems with applications in biology. Ph.D. thesis, Department of Mathematical Sciences, Division of Mathematics, Chalmers University of Technology and Gbteborg University Gbteborg,Sweden 2007.